Complete the square. $\left(\frac{b}{2a}\right)$

$ax^2 + bx + c = 0$ $\quad \frac{-4ac}{4a^2}$

$ax^2 + bx = -c$

$x^2 + \frac{b}{a}x + \frac{b^2}{4a^2} = -\frac{c}{a} + \frac{b^2}{4a^2}$

$\sqrt{\left(x + \frac{b}{2a}\right)^2} = \sqrt{\frac{b^2 - 4ac}{4a^2}}$

$x + \frac{b}{2a} = \frac{\pm\sqrt{b^2-4ac}}{2a}$

$x = \frac{-b \pm \sqrt{b^2-4ac}}{2a}$

6-5 The Quadratic Formula and the Discriminant

$ax^2 + bx + c = 0$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

ex 1

$3x^2 + x - 1 = 0$

$a = 3$
$b = 1$
$c = -1$

$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$x = \frac{-1 \pm \sqrt{1 - 4(3)(-1)}}{2(3)}$

$x = \frac{-1 \pm \sqrt{13}}{6}$

ex 2

$5x^2 + 8 = -12x$

$5x^2 + 12x + 8 = 0$

$a = 5$
$b = 12$
$c = 8$

$x = \frac{-12 \pm 4i}{10}$

$x = \frac{-6 \pm 2i}{5}$

The Discriminant

$D = b^2 - 4ac$ $\quad x = \frac{-b \pm \sqrt{D}}{2a}$

Determines the nature of the roots.

Three Cases

I. D > 0 2 ℝ unequal roots

II. D = 0 double ℝ root

III. D < 0 2 imaginary roots

Ex

$x^2 - 8x + 5 = 0$

Use D to determine the nature of roots

$a = 1$
$b = -8$
$c = 5$

$D = 64 - 4(1)5$
$D = 44$

2 ℝ roots irrational

Also able to determine if the roots are rational or irrational.

Rational
a, b, & c must be rational and D must be a perfect square (Real)

ex

$x^2 + 10x + 25 = 0$

$a = 1$
$b = 10$
$c = 25$

$D = b^2 - 4ac$
$100 - 4(25$
$D = 0$

double ℝ root, rational

ex

$x^2 - 4x + 13 = 0$

$a = 1$
$b = -4$
$c = 13$

$D = 16 - 4(13)$

2 imaginary roots

Determine as much as you can about the roots:

13 1. $y^2 - 3y - 1 = 0$ 2 ℝ irrat.

-32 2. $3a^2 - 10a = -11$ 2 imaginary

0 3. $5x^2 + 2\sqrt{10}x + 2 = 0$ double ℝ, irrat.

484 4. $3b^2 = 14b + 24$ 2 ℝ rat.

$196 - 4(3)(-24)$
$196 + 288$

Find the value for k such that there are 2 imaginary roots

ex

$5x^2 - 2x + k = 0$

$D < 0$ $4 - 4(5)k < 0$
$4 - 20k < 0$
$4 < 20k$
$\frac{1}{5} < k$

Find the value for k such that there is a double root

ex

$3x^2 + 2x + k = 0$

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15-27 odd and 16, 18